

# Bouncing Galileon Cosmologies

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We present nonsingular, homogeneous and isotropic bouncing solutions of the conformal Galileon model. We show that such solutions necessarily begin with a radiation-dominated contracting phase. This is followed by a quintom scenario in which the background equation of state crosses the cosmological constant boundary allowing for a nonsingular bounce which in turn is followed by Galilean Genesis. We analyze the spectrum of cosmological perturbations in this background. Our results show that the fluctuations evolve smoothly and without any pathology, but the adiabatic modes form a blue tilted spectrum. In order to achieve a scale-invariant primordial power spectrum as required by current observations, we introduce a light scalar field coupling to the Galileon kinetically. We find two couplings which yield a scale-invariant spectrum, one of which requires a fine tuning of the initial conditions. This model also predicts a blue tilted spectrum of gravitational waves stemming from quantum vacuum fluctuations in the contracting phase.

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## I. INTRODUCTION

Nonsingular bouncing cosmologies avoid the cosmological “Big Bang” singularity of Standard Cosmology and often solve the horizon problem, thus they have attracted a lot of attention in the 80 years since their introduction in Ref. [1]. They have been studied in models motivated by approaches to quantum gravity such as modified gravity models [2–5], Lagrangian multiplier gravity actions (see e.g. [6, 7]), non-relativistic gravitational actions [8, 9], brane world scenarios [10, 11], torsion gravity [12], “Pre-Big-Bang” [13] and Ekpyrotic [14] cosmology (in which case the conjectured bounces are classically singular) and loop quantum cosmology [15]. Bouncing cosmologies have also been conjectured to occur in some string models. For example, it may be possible to embed the String Gas Cosmology [16] in a bouncing universe [17]. Non-singular bounces may also be studied using effective field theory techniques by introducing matter fields that violate certain energy conditions, for example non-conventional fluids [18, 19], double-field quintom matter [20, 21], fields non-minimally coupled to gravity [22] and ghost condensates [23–25]. A non-singular bounce may also arise in a universe with an open spatial curvature term (see e.g. [26, 27]). A specific realization of a quintom bounce was obtained in the Lee-Wick cosmology [28], see also [29] for its instability. Various bouncing models were reviewed in Ref. [30].

A successful non-singular bouncing cosmology is typically accompanied by a violation of the null energy condition (NEC) in a neighborhood of the bounce [20]. The NEC, which states that  $T_{\mu\nu}n^\mu n^\nu \geq 0$  for any null vector  $n^\mu$ , implies that the Hubble parameter of a Friedmann-Robertson-Walker (FRW) universe is always decreasing throughout the cosmic evolution. However, a non-singular bounce requires that the time derivative of the Hubble parameter satisfies  $\dot{H} > 0$  when  $H = 0$  so that the universe is able to transit from a contracting phase to an expanding one. Moreover, in order to link a stable contraction with a normal thermal expanding history as observed in our universe, the equation of state (EoS) of the background universe has to cross the cosmological constant boundary, which corresponds to the so-called quintom scenario originally discovered in the context of dark energy phenomenology [31]. Consistent models realizing quintom scenarios are notoriously difficult to construct. For example in models described by a single perfect fluid or a single scalar with a Lagrangian of k-essence form [32], the cosmological perturbations encounter a divergence when the NEC is violated [31, 33–36]. This statement was explicitly proven in Ref. [37] as a “No-Go” theorem for dynamical dark energy models. Furthermore in the absence of higher derivative terms it was demonstrated that any

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violation of NEC in a very general class of models necessarily leads to either instability or superluminal propagation [38].

To realize a viable quintom scenario of cosmological evolution in a theory of a single scalar field, one needs to introduce unconventional operators in the Lagrangian. The simplest and also the first quintom model was constructed by a combination of a canonical scalar and a ghost [31], which has a property of cosmic duality [39]. Later on, it was realized that the EoS of a scalar can cross  $-1$  by introducing higher order derivative terms [40–42]. However in such models, there is a quantum instability due to an unbounded vacuum state [43]. A possible approach to stable violations of the NEC is the ghost condensation of Ref. [44], in which the negative kinetic modes are bounded via a spontaneous Lorentz symmetry breaking, although it might allow for superluminal propagation of information in some cases [38]. Various theoretical realizations of quintom scenarios and their implications for early universe physics were reviewed in Ref. [45] (see also [46]).

Recently, beginning with Ref. [47] a class of models which stably violates the NEC has been studied extensively. These are models of a single scalar field called the Galileon. They are local infrared modifications of general relativity, generalizing an effective field description of the DGP model [48]. Various Galileon configurations which violate NEC have been shown to be stable, but generically such configurations admit superluminal propagation [49] and perhaps even closed timelike curves [50]. The key feature of these models is that they contain higher order derivative terms in the action while the equation of motion remains second-order in order to avoid the appearance of ghost modes, realizing the idea pioneered by Horndeski thirty years ago [51]. The scalar sector of the theory enjoys a particular realization of either Galilean or conformal Galilean symmetry, although this is not respected by ghost-free couplings to gravity [52]. Later on, this model was generalized to a DBI version [53], the K-Mouflage scenario [54], the supersymmetric Galileon [55], the Kinetic Gravity Braiding models [56], the generic Galileon-like action [51, 57, 58] and others. Their phenomenology has been studied extensively, for example in Refs. [59–68].

A class of solutions of the conformal Galileon model which interpolates between a Minkowski initial state and a final big rip singularity was described in Ref. [69]. These solutions are stable and in fact are dynamical attractors. The authors noted that, as the energy diverges, the effective Galilean description will break down before the big rip singularity and so this scenario may provide a new and violent alternative to inflation which solves the initial singularity and horizon problems. They referred to this scenario as “Galilean Genesis” and found that its evolution is characterized by a strong but stable violation of the NEC yielding a short duration of fast growth of the background energy density. After the Galilean Genesis, other degrees of freedom become dominant and the universe can be reheated through a defrosting process by virtue of a coupling of matter to the Galileon field [70].

However one does not need to assume that the initial state is finely tuned to be a Minkowski universe. In this note we will show that generic Galilean Genesis cosmologies begin with a radiation-dominated contraction, followed by a quintom scenario and a nonsingular bounce. This bounce is unusual in several respects. First, the usual motivations for bounces, the singularity and the horizon problem, do not apply. The Galilean Genesis scenario already is nonsingular and solves the horizon problem. However, the fact that the bounce leads into a dynamical attractor means that the analysis of the inhomogeneity problem associated with bouncing cosmologies will certainly be different, and perhaps our bounce will be more phenomenologically viable. In this background, the cosmological perturbations of scalar type and tensor type evolve smoothly and controllably, but as in the usual Galilean Genesis model are unable to form scale-invariant spectra to explain the current cosmological observations. As in that case, one may introduce another light scalar coupled to the Galileon, which plays a role of a curvaton [71–75] in the bounce model [76]. By correctly choosing the couplings, it is possible to produce a nearly scale-invariant primordial power spectrum. We consider a kinetic coupling of the curvaton to the Galileon. Our results show that there exist two couplings which generate a scale-invariant power spectrum. One of these couplings features stable curvaton dynamics. If one instead uses the second coupling, the backreaction grows quickly in the contracting phase and so the model requires a fine-tuned initial condition. Note that, similar kinetic couplings have been discussed in Refs. [77–80].

We begin in Section II with a quick review of the conformal Galileon model coupled to gravity and its cosmology. In Section III we analytically determine the asymptotic behavior of a generic nonsingular bounce solution and we numerically solve for the entire evolution. Section IV is devoted to a study of the cosmological perturbations. The analysis of the curvature perturbations shows that the evolution of these perturbations is pathology-free, even during the quintom phase. In order to achieve a scale-invariant primordial power spectrum, we study the curvaton bounce mechanism with a generalized kinetic coupling to the Galileon field in Section V. Section VI presents a summary and discussion.

We use the convention  $M_{pl} = 1/\sqrt{8\pi G}$  with  $G$  Newton’s gravitational constant. The signature of the metric is chosen to be  $(-, +, +, +)$ .

## II. GALILEON COSMOLOGY BASICS

For concreteness we consider the simplest model of a conformal Galileon scalar field  $\Pi$  (which in our notation is dimensionless) minimally coupled to Einstein gravity. The Lagrangian for  $\Pi$  consists of an unusual kinetic term coupling of  $\Pi$  to itself, a nonlinear term similar to that of the DGP model, and a square of the kinetic term which is required by conformal symmetry [47, 69]

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + F^2 e^{2\Pi} (\partial\Pi)^2 + \frac{F^3}{M^3} (\partial\Pi)^2 \square\Pi + \frac{F^3}{2M^3} (\partial\Pi)^4 \right]. \quad (1)$$

Here  $R$  is the Ricci scalar of the 4-dimensional space-time, the coefficient  $F$  is the mass scale of the Galileon field, and  $M$  is another mass scale which suppresses the higher derivative operators. The Lorentz indices are contracted with the metric  $g_{\mu\nu}$  and the box operator is built from covariant derivatives:  $\square \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$ .

We note that, when the derivatives are large, neither the gravitational sector [81, 82] nor the Galileon sector [47, 50] is captured by this action. This action describes a low energy effective field theory. In our bouncing cosmologies the energy density shrinks to zero at early times, but diverges as one approaches the Galilean Genesis. Thus the effective field theory description is reliable except within some neighborhood of the Genesis where, as in [70], one may make use of an entropy field to reheat the universe so that the potential strong coupling problem of Galileon cosmology can be avoided<sup>1</sup>.

By varying the action with respect to the inverse metric, one obtains the stress energy tensor

$$\begin{aligned} T_{\mu\nu} = & -F^2 e^{2\Pi} [2\partial_\mu \Pi \partial_\nu \Pi - g_{\mu\nu} (\partial\Pi)^2] \\ & - \frac{F^3}{M^3} [2\partial_\mu \Pi \partial_\nu \Pi \square\Pi - \partial_\mu \Pi \partial_\nu (\partial\Pi)^2 - \partial_\nu \Pi \partial_\mu (\partial\Pi)^2 + g_{\mu\nu} \partial_\sigma \Pi \partial^\sigma (\partial\Pi)^2] \\ & - \frac{F^3}{2M^3} (\partial\Pi)^2 [4\partial_\mu \Pi \partial_\nu \Pi - g_{\mu\nu} (\partial\Pi)^2]. \end{aligned} \quad (2)$$

We are interested in the cosmological evolution of a flat FRW universe

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2 \quad (3)$$

in the conformal Galileon model. Substituting the metric (3) into Einstein's equations, we obtain the Friedmann equations:

$$H^2 = \frac{8\pi G}{3} \rho, \quad (4)$$

$$\dot{H} = -4\pi G(\rho + P), \quad (5)$$

where  $H$  is the Hubble parameter characterizing the evolution of the universe. The energy density and the pressure are

$$\rho = F^2 [-e^{2\Pi} \dot{\Pi}^2 + \frac{1}{\bar{H}^2} (\dot{\Pi}^4 + 4H\dot{\Pi}^3)] , \quad (6)$$

$$P = F^2 [-e^{2\Pi} \dot{\Pi}^2 + \frac{1}{3\bar{H}^2} (\dot{\Pi}^4 - 4\dot{\Pi}^2 \ddot{\Pi})] , \quad (7)$$

respectively, where we have introduced the constant

$$\bar{H} \equiv \sqrt{\frac{2M^3}{3F}}. \quad (8)$$

Inserting the energy density (6) into the first Friedmann equation (4), one easily solves for Hubble's constant

$$H = 2\alpha \dot{\Pi}^3 \pm \sqrt{-\alpha \bar{H}^2 e^{2\Pi} \dot{\Pi}^2 + \alpha \dot{\Pi}^4 + 4\alpha^2 \dot{\Pi}^6}, \quad (9)$$

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<sup>1</sup> A generic scenario for preheating in a bouncing cosmology was discussed in Ref. [83], where it was shown that a period of stochastic resonance efficiently takes a universe from a primordial era to the radiation phase.

where

$$\alpha \equiv \frac{F^2}{3M_p^2 \bar{H}^2} = \frac{F^3}{2M^3 M_p^2} \quad (10)$$

is a constant with dimensions of area. Note that the reality of the square root of the above expression (9) implies that if  $\dot{\Pi}$  is nonzero, then its magnitude must be greater than the strictly positive and  $\Pi$ -dependent value,

$$\dot{\Pi}^2 \geq \frac{1}{8\alpha} (\sqrt{1 + 16\alpha \bar{H}^2 e^{2\Pi}} - 1) , \quad (11)$$

which means that  $\dot{\Pi}$  can never cross zero and so the Galileon scalar  $\Pi$  is either a constant or monotonic. Therefore the space of solutions is not connected. In particular the Galilean Genesis inhabits the monotonically increasing branch while a Galilean Apocalypse inhabits a monotonically decreasing branch. This clarifies the claim in Ref. [69] which, based on an analysis of linear perturbations, concluded that initially shrinking universes may be in the domain of attraction of a Galilean Genesis. These initially shrinking universes nevertheless have a monotonically increasing Galileon field. As we are interested in configurations which lead to a Galilean Genesis, we will only consider the branch in which  $\Pi$  monotonically increases.

### III. GALILEON BOUNCE BACKGROUND SOLUTIONS

In this section, we will describe bouncing solutions in Galileon cosmology. Recall that a nonsingular bounce requires  $\dot{H} > 0$  when  $H = 0$  [20]. In Eq. (9), the first term of the Hubble parameter is positive as  $\Pi$  is increasing. Therefore, a change of sign of the Hubble parameter in Galileon cosmology can only occur if the second term of (9) is negative. Therefore the negative branch of Eq. (9) is needed to obtain a bouncing solution.

#### A. Asymptotic Solution

Eq. (9) of the previous section determines the Hubble parameter  $H$  as a function of the Galileon field  $\Pi$  and its first derivative. According to the Friedmann equation (5), the Hubble parameter is also determined by the energy density and the pressure of the Galileon field. Therefore one may use (9) to eliminate the Hubble parameter from Eqs. (5) and (6) thus obtaining a second order differential equation for the Galileon field  $\Pi$  alone

$$4\dot{\Pi}^2 \ddot{\Pi} - 3\bar{H}^2 e^{2\Pi} \dot{\Pi}^2 + 2\dot{\Pi}^4 + 12\alpha \dot{\Pi}^6 = 6\dot{\Pi}^3 \sqrt{B} + \frac{\dot{B}}{2\alpha \sqrt{B}} , \quad (12)$$

where we have defined  $B$  to be

$$B = -\alpha \bar{H}^2 e^{2\Pi} \dot{\Pi}^2 + \alpha \dot{\Pi}^4 + 4\alpha^2 \dot{\Pi}^6 , \quad (13)$$

which is just the quantity in the square root in Eq. (9).

At the moment of the bounce  $H = 0$  and so using Eq. (9) and the positivity of  $\dot{\Pi}$

$$\dot{\Pi} = \bar{H} e^{\Pi} . \quad (14)$$

On the other hand, before the bounce  $H < 0$  and so (9) implies

$$\dot{\Pi} > \bar{H} e^{\Pi} . \quad (15)$$

In this subsection we will be interested in the behavior of  $\Pi$  and the spacetime geometry in the asymptotic past, therefore the condition (15) will be satisfied.

This leads us to two possibilities far in the past. Either  $\dot{\Pi}$  becomes so much larger that the  $\bar{H} e^{\Pi}$  terms can be neglected, or else the ratio of these terms tends to a constant.<sup>2</sup> We will now show that the second possibility is inconsistent.

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<sup>2</sup> In principle, the ratio may tend to 0 but the smaller term may still be important due to a cancelation, however in this case the equations of motion without the exponential term would lead to a solution at leading order which either fails or is ambiguous at successive orders. We will see that this is not the case.

Imagine that in the asymptotic past the ratio indeed tended to a constant  $c_0$

$$\dot{\Pi} = c_0 \bar{H} e^{\Pi} . \quad (16)$$

We know from Eq. (15) that  $c_0 \geq 1$ . We can easily solve Eq. (16) to find

$$\Pi = -\ln(c_0 \bar{H}(t_0 - t)) , \quad \dot{\Pi} = \frac{1}{t_0 - t} \quad (17)$$

for some constant of integration  $t_0$ . If  $c_0 = 1$  then (12) yields the original Galilean Genesis solution, which violates (15) and so  $c_0 > 1$ . Therefore the first two terms in the expression (13) for  $B$  are both of order  $1/t^4$ , and since  $c_0 \neq 1$  they do not cancel. This implies that the last term on the right hand side of the equation of motion (12) is of order  $1/t^3$ , whereas all other terms are of higher order in  $1/t$ . Thus the equation of motion (12) has no solution as  $t \rightarrow -\infty$ , and we have proved a contradiction. So no such constant  $c_0$  exists, and in the far past

$$\dot{\Pi} \gg \bar{H} e^{\Pi} . \quad (18)$$

Now the equation of motion simplifies slightly to

$$4\dot{\Pi}^2 \ddot{\Pi} + 2\dot{\Pi}^4 (6\alpha \dot{\Pi}^2 + 1) = 6\dot{\Pi}^5 \sqrt{4\alpha^2 \dot{\Pi}^2 + \alpha} + \frac{12\alpha \dot{\Pi}^3 + 2\ddot{\Pi}}{\sqrt{4\alpha^2 \dot{\Pi}^2 + \alpha}} \ddot{\Pi} . \quad (19)$$

Inspecting Eq. (19) one sees that the dimensionless quantity  $\alpha \dot{\Pi}^2$  appears frequently. Therefore the behavior of the solution depends strongly on whether  $\Pi$  grows more rapidly than the constant scale  $1/\sqrt{\alpha}$ . We can again consider three cases, corresponding to one scale being much larger and to them being comparable.

- **Case 1: Fast roll**

First let us consider the case in which in the distant past

$$\alpha \dot{\Pi}^2 \gg 1 . \quad (20)$$

Then the equation of motion simplifies to

$$\frac{1}{2} \dot{\Pi}^4 = 2\ddot{\Pi}^2 \ddot{\Pi} . \quad (21)$$

Clearly  $\dot{\Pi} = 0$  does not satisfy the condition (20), therefore (21) further simplifies to

$$\frac{1}{2} \dot{\Pi}^2 = 2\ddot{\Pi} . \quad (22)$$

This equation implies that in the far past  $\dot{\Pi}$  is asymptotic to a constant multiplied by  $1/t$  and therefore goes to zero, violating the hypothesis (20). This hypothesis is therefore inconsistent.

- **Case 2: Linear evolution**

The next possibility is that  $\alpha \dot{\Pi}^2$  tends to a constant, in which case  $\dot{\Pi}$  tends to a constant while  $\ddot{\Pi}$  tends to zero. The latter condition implies that only the second term on the left side and first on the right side of Eq. (19) survives the infinite past limit. Squaring both sides, the  $\dot{\Pi}^{12}$  terms cancel. The positivity of  $\dot{\Pi}^2$  then implies that the left hand side is greater, and so this equation cannot be satisfied, and again the hypothesis that  $\dot{\Pi}$  tends to a constant is inconsistent.

- **Case 3: Slow roll**

This leaves a single case to be considered

$$\alpha \dot{\Pi}^2 \ll 1 . \quad (23)$$

In this case the dominant terms in the equation of motion are

$$2\dot{\Pi}^4 = \frac{2}{\sqrt{\alpha}} \dot{\Pi} \ddot{\Pi} . \quad (24)$$

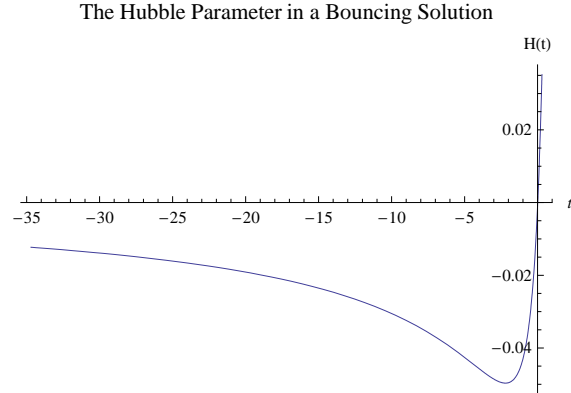


FIG. 1: The Hubble parameter in a bouncing solution first shrinks as in a radiation dominated phase, then crosses zero during the nonsingular bounce and eventually evolves to the moment of Galilean Genesis. In the numerical calculation, the values of parameters are listed in (29).

Eliminating the constant solution branch we are left with

$$\ddot{\Pi} = \sqrt{\alpha} \dot{\Pi}^3 \quad (25)$$

which is solved by

$$\dot{\Pi} = \frac{1}{\alpha^{1/4} \sqrt{2(t_0 - t)}} \quad (26)$$

for an arbitrary time  $t_0$ . Integrating this leads to the general asymptotic solution

$$\Pi = \Pi_0 - \frac{\sqrt{2(t_0 - t)}}{\alpha^{1/4}}. \quad (27)$$

Our second order differential equation (12) led to two constants of integration,  $t_0$  and  $\Pi_0$ . While choices of the constant of integration  $t_0$  are related by a time-translation symmetry, choices of  $\Pi_0$  are related by a shift-symmetry of  $\Pi$  which is broken by the exponential term. This symmetry appears only in the far past where the exponential term is negligible. Therefore the constant of integration  $\Pi_0$  labels physically inequivalent solutions. The above argument implies that this single parameter family of solutions contains all nonsingular, homogeneous and isotropic bouncing solutions of this conformal Galileon model. We find bouncing solutions for all positive values of  $\Pi_0$ . As  $\Pi_0$  becomes increasingly negative the bounces become increasingly violent and short, for example at  $\Pi_0 = -35$  we find that the scale factor of the universe changes by a factor of 2 in a time .0001 as measured in the natural units of Eq. (29). Beyond this threshold, the gradients become so steep that the problem is no longer amenable to a numerical treatment, and probably the effective low energy description breaks down. In this paper, for concreteness, we will consider the solution  $\Pi_0 = 0$ , commenting on other cases when they differ qualitatively. As  $\Pi_0$  is subdominant in the far past, where cosmological perturbations are generated, this arbitrary choice will have little effect on our main results.

One can now use Eq. (9) to determine the asymptotic evolution of the space-time. To leading order as  $t$  tends to  $-\infty$ , one finds

$$H = \frac{1}{2(t - t_0)}. \quad (28)$$

As desired this is negative, the universe is contracting. The coefficient identifies it as a radiation dominated phase, and as we will review momentarily, numerically we have verified that the equation of state in the far past of a bounce solution indeed tends to  $w = 1/3$ .

## B. Numerical computation

In the previous subsection we have found that there exists a family of potential bouncing solutions. However, recall that a bouncing solution requires that the Hubble constant be initially less than zero and thus the inequality

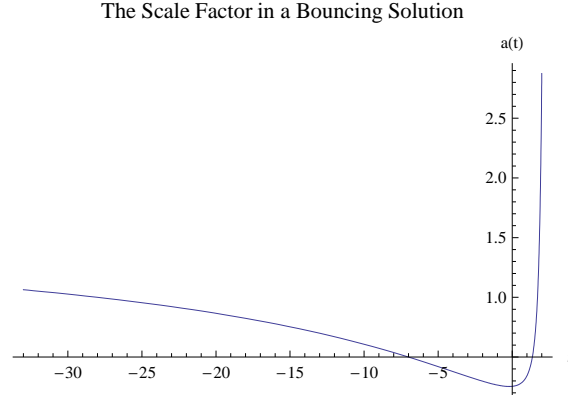


FIG. 2: The scale factor  $a(t)$  in a bouncing solution first shrinks in a radiation dominated phase, then arrives at a nonzero minimal value at the bouncing point and after that enters an expanding phase. In the numerical calculation, the values of parameters are listed in (29).

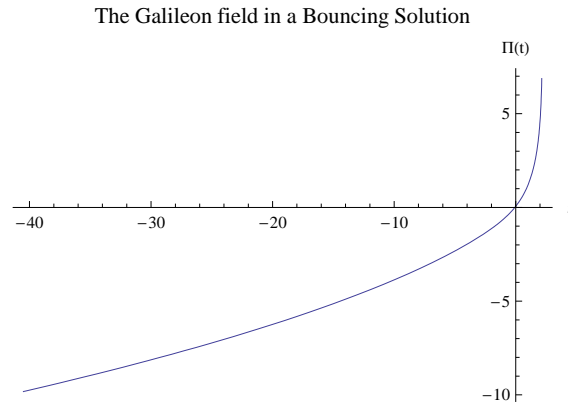


FIG. 3: The Galileon  $\Pi$  monotonically increases throughout the whole cosmological evolution in the model of Galileon bounce. In the numerical calculation, the values of parameters are listed in (29).

(15) places an upper bound on the Galileon  $\Pi$  at any finite value of the cosmic time  $t$ . Therefore the existence of a bouncing solution is not guaranteed. We must rely upon a numerical analysis, whose results we will now summarize.

For concreteness in the numerical calculation, we set

$$F = \bar{H} = 1, \quad M_p^2 = \frac{1}{8\pi G} = 1, \quad \alpha = \frac{1}{3}, \quad (29)$$

and we also make time translation  $t \rightarrow t - t_0$  in the numerical plots in order to remove the meaningless constant  $t_0$ . We found that the Hubble radius indeed begins as in a radiation dominated collapse and then becomes positive during the bounce, ending in a rapidly expanding Galilean Genesis as seen in Fig. 1. The corresponding scale factor  $a(t)$  can be seen in Fig. 2. The Galileon  $\Pi$ , as expected, increases monotonically throughout the evolution as shown in Fig. 3. We can see that  $\Pi$  becomes positive near the moment of the bounce. Therefore, in the expanding phase the factor  $e^{2\Pi}$  in the action is no longer negligible.

To illustrate that the contracting universe is indeed in a radiation dominated phase, we plot the evolution of the EoS parameter  $w \equiv P/\rho$ , in Fig. 4. Indeed, we can easily observe that  $w \rightarrow 1/3$  in the far past, then it crosses the cosmological constant boundary  $-1$  and falls down toward negative infinity, which corresponds to the moment of Galilean Genesis. This part of the numerical computation demonstrates the quintom scenario indeed is realized in the bounce model of Galileon cosmology.

Before ending this subsection, we would like to make a quick comment on how possible it will be for a bounce to happen, for different choice of initial conditions and parameters such as  $F$  and  $M$ . Since the condition for bounce is to let  $H = 0$  where  $H$  is given by Eq. (9), and we can see that in (9)  $\Pi$  only appears in the index of exponential term,

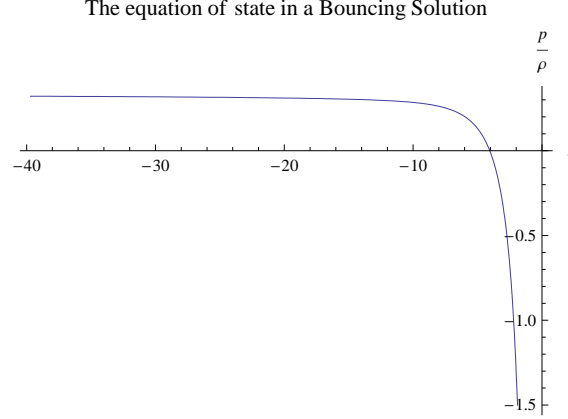


FIG. 4: The ratio of the pressure to the density of the Galileon field begins at  $1/3$ , which is the same as that of normal radiation. It steadily decreases and crosses  $w = -1$  just before the bounce. In the numerical calculation, the values of parameters are listed in (29).

so as long as we require  $\Pi$  be negative, this term will be negligible comparing to other terms, and different value of  $\Pi$  will not be sensitive for a bounce to happen. The initial value of  $\dot{\Pi}$  and the other two parameters will, however, play an important role in deciding whether bounce or not, so more or less some fine tuning might be needed. This could be seen as the price to construct a successful and healthy bounce model.

#### IV. COSMOLOGICAL PERTURBATIONS AND PRIMORDIAL POWER SPECTRUM

We have completed our analysis of homogeneous and isotropic bouncing solutions. Now we will consider the evolution of cosmological perturbations on these backgrounds. In particular we will see that our bouncing solution is stable and free of ghost instabilities throughout its evolution, even during the quintom and bouncing phases. At linear order, in standard General Relativity, cosmological perturbations can be decomposed into scalar and tensor types [84] which evolve separately. In Galileon cosmology the equation of motion for the scalar field is still of second order and the symmetries of our solution and action are still those of the FRW universe and general relativity respectively. Therefore we expect that the decomposition of cosmological perturbations in Galileon cosmology is similar to that in General Relativity.

A convenient method of studying cosmological perturbation is to work with the Arnowitt-Deser-Misner (ADM) decomposition of the metric [85],

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (30)$$

where  $N(x)$  and  $N_i(x)$  are the lapse function and the shift vector, respectively. The tensor  $h_{ij}$  is the metric of 3-dimensional space. We can decompose the action (1) into the 3 + 1 form

$$\mathcal{S} = \int dt d^3x \sqrt{h} \frac{N}{2} \left[ M_p^2 (R^{(3)} + K_{ij} K^{ij} - K^2) - 4F^2 e^{2\Pi} X - \frac{4F^3}{M^3} X \square \Pi + \frac{4F^3}{M^3} X^2 \right], \quad (31)$$

where we have defined

$$\begin{aligned} X &\equiv -\frac{1}{2}(\partial\Pi)^2 \\ &= \frac{1}{2N^2}(\dot{\Pi} - N_j h^{ij} \partial_i \Pi)^2 - \frac{1}{2} h^{ij} \partial_i \Pi \partial_j \Pi. \end{aligned} \quad (32)$$

In the above equation,  $R^{(3)}$  is the Ricci scalar of a 3-dimensional time-slice.  $K_{ij}$  is the corresponding extrinsic tensor

$$K_{ij} \equiv \frac{1}{2N}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i), \quad (33)$$

where  $\nabla_i$  is the covariant derivative constructed using the spatial metric  $h_{ij}$  and whose indices are raised and lowered with  $h_{ij}$ .



### A. Scalar perturbations

First, we consider scalar cosmological perturbations. We will work in the uniform  $\Pi$  gauge, in which the perturbations of the Galileon scalar and the metric are given by

$$\begin{aligned}\delta\Pi &= 0, \\ h_{ij} &= a^2 e^{2\zeta} \delta_{ij}.\end{aligned}\tag{34}$$

Varying the action (31) with respect to the lapse function  $N$  and the shift vector  $N_i$  respectively yields the following Hamiltonian and momentum constraint equations:

$$M_p^2(R^{(3)} - K_{ij}K^{ij} + K^2) + 2F^2 e^{2\Pi} \frac{\dot{\Pi}^2}{N^2} - \frac{3F^3}{M^3} \frac{\dot{\Pi}^4}{N^4} - \frac{4F^3}{M^3} \frac{\dot{\Pi}^3}{N^4} \frac{\sqrt{h}}{\sqrt{h}} + \frac{4F^3}{M^3} \frac{\dot{\Pi}^3}{N^4} \frac{\sqrt{h}_{,i}}{\sqrt{h}} N^i + \frac{4F^3}{M^3} \frac{\dot{\Pi}^3}{N^4} N^i_{,i} = 0, \tag{35}$$

$$\partial_j K_i^j + \bar{\Gamma}_{jl}^j K_i^l - \bar{\Gamma}_{ji}^l K_l^j - \partial_i K - \frac{2F^3}{M^3 M_p^2} \frac{\dot{\Pi}^3}{N^4} \partial_i N = 0. \tag{36}$$

We expand  $N$  and  $N_i$  to first order

$$N = 1 + \varphi, \quad N_i = \partial_i \psi. \tag{37}$$

The constraint equations yield

$$\varphi = \frac{M_p^2 \dot{\zeta}}{M_p^2 H - (F/M)^3 \dot{\Pi}^3} \tag{38}$$

and

$$\psi = \frac{M_p^2 \dot{\zeta}}{(F/M)^3 \dot{\Pi}^3 - M_p^2 H} + \frac{a^2 [3(F/M)^6 \dot{\Pi}^6 - M_p^2 F^2 e^{2\Pi} \dot{\Pi}^2 + 3(F/M)^3 M_p^2 \dot{\Pi}^4 + 9(F/M)^3 M_p^2 \dot{\Pi}^3 H]}{[(F/M)^3 \dot{\Pi}^3 - M_p^2 H]^2} \partial^{-2} \dot{\zeta}. \tag{39}$$

Making use of these expressions for  $\varphi$  and  $\psi$ , one can expand the action up to second order,

$$\mathcal{S}_2 = 3 \int dt d^3 x a^3 D M_p^2 [\dot{\zeta}^2 - \frac{c_s^2}{a^2} (\partial_i \zeta)^2], \tag{40}$$

where the dimensionless factor  $D$  in front of the time derivative term is

$$D = \frac{2M_p^4 H^2 + 2(F/M)^6 \dot{\Pi}^6 + (F/M)^3 M_p^2 \dot{\Pi}^4}{2[M_p^2 H - (F/M)^3 \dot{\Pi}^3]^2}, \tag{41}$$

and the sound speed squared is

$$c_s^2 = \frac{-2M_p^4 \dot{H} + 2(F/M)^3 M_p^2 H \dot{\Pi}^3 - 2(F/M)^6 \dot{\Pi}^6 + 6(F/M)^3 M_p^2 \dot{\Pi}^2 \ddot{\Pi}}{3[2M_p^4 H^2 + (F/M)^3 M_p^2 \dot{\Pi}^4 + 2(F/M)^6 \dot{\Pi}^6]}. \tag{42}$$

We note that a similar but more generic analysis of the perturbation theory of Galileon cosmology was performed in Ref. [86] and expressions for  $D$  and  $c_s^2$  were also obtained in Ref. [56]. Our results (41) and (42) are consistent with the corresponding general calculations in Refs. [56, 86] specialized to the conformal Galileon theory. From Eq. (41), we can easily observe that the factor  $D$  is positive definite, and so there do not exist any modes carrying negative kinetic energy in this background. Consequently, we can conclude that the Galileon bounce does not suffer from a ghost instability.

The sound speed  $c_s$  characterizes the propagation of a fluctuation mode with a fixed comoving wavelength and thus determines the gradient stability. If the numerator of the sound speed squared (42) is nonnegative, then this model is also well-behaved upon the propagation of its perturbation modes. However, the determination of the sound speed parameter is highly solution-dependent. Therefore, we will study the propagation of the scalar perturbation modes in detail.

We numerically plot the evolutions of the parameters  $D$  and  $c_s^2$  in Figs. 5 and 6, respectively. From these two figures, we can see that both parameters are positive definite throughout the contracting phase and the bouncing phase.

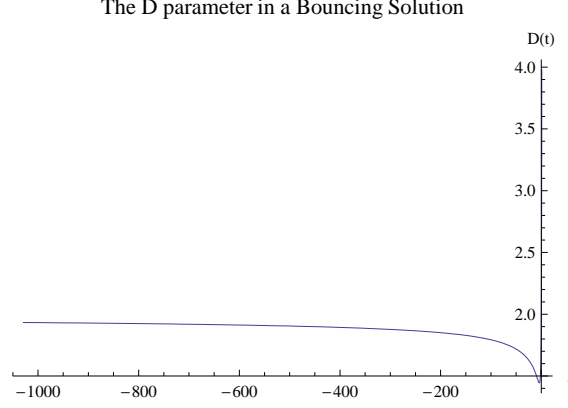


FIG. 5: Plot of the parameter  $D$  in the context of Galileon bounce. The background parameters are chosen to be the same as the values provided in Eq. (29).

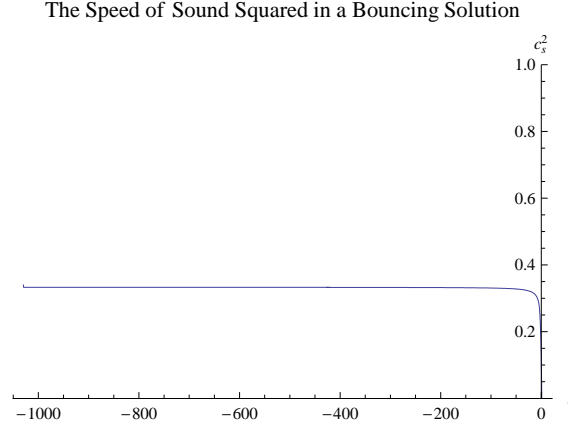


FIG. 6: Plot of the parameter  $c_s^2$  in the context of Galileon bounce. The background parameters are chosen to be the same as the values provided in Eq. (29).

Moreover, the value of  $c_s^2$  is less than 1, which implies there is no superluminal propagation for the perturbation modes, and so no risk of closed timelike curves so long as the perturbations remain small enough to be described by this linear analysis. At higher values of  $\Pi_0$  there is a bump in  $c_s$  around the time of the bounce, whose height grows with  $\Pi_0$ , however the maximum velocity falls just short of the speed of light<sup>3</sup>. Thus small perturbations about our background are causal.

After the bouncing phase, the universe will approach the moment of Galilean Genesis. Here  $D$  diverges and  $c_s^2$  briefly passes below zero, as seen in Fig. 7. The negative value of  $c_s^2$  indicates an instability in the model, however as has been stressed in Ref. [69] the high energy density implies that perturbative results obtained from the low energy effective Galileon theory are not to be trusted in this region. In that note the authors were unable to characterize the scale at which the low energy effective Galileon description breaks down, arguing merely that this occurs before the Planck scale. Here however the negativity of  $c_s^2$  very close to the Galilean genesis solution provides a novel upper bound on the latest time at which the Galileon effective theory is reliable.

We note however that when  $\Pi_0 \lesssim -3$ ,  $c_s^2$  dips below zero before the bounce. This suggests that bouncing solutions are only stable for sufficiently large initial values of the Galileon  $\Pi$ . This is not so surprising, in light of the aforementioned observation that at very low values of  $\Pi_0$  the time derivatives become very large even before the bounce

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<sup>3</sup> Indeed in the high  $\Pi_0$  limit there is a long, slow bounce during which the universe is essentially Minkowski. In this sense, the usual Galilean Genesis cosmology can be considered to be the large  $\Pi_0$  limit of our scenario.

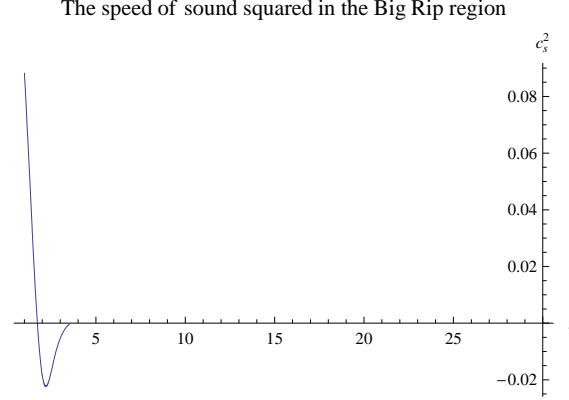


FIG. 7: Long after the bounce, during the Galilean Genesis, the speed of sound squared dips below zero. Naively this indicates a gradient instability, however the Galileon effective theory cannot be trusted in the high energy density regime close to the big rip singularity. In the numerical calculation, the values of parameters are listed in (29).

and so one expects a premature breakdown of the effective theory in this parameter range. Therefore we learn that a Galilean genesis can only occur if the initial condition  $\Pi_0$  exceeds a critical value of about  $-3$ .

In order to provide a well-defined quantization for the cosmological perturbations, it is usually convenient to redefine the perturbation variable so that the second order action is of canonical form. In the case of Galileon cosmology, we introduce the variable

$$u \equiv z\zeta, \quad z \equiv a\sqrt{D}. \quad (43)$$

The momentum space equation of motion for a mode of this new variable is

$$u_k'' + (c_s^2 k^2 - \frac{z''}{z})u_k = 0, \quad (44)$$

where the prime denotes the derivative with respect to the conformal time  $\eta \equiv \int a^{-1}(t)dt$ .

Recall that, from the study of background dynamics, one has learned that the universe is radiation-dominated during the contracting phase. Thus it is easy to find  $a(\eta) \sim \eta$  and so  $a'' = 0$ . Making use of this relation, one can write the perturbation equation in a Klein-Gordon form

$$u_k'' + (c_s^2 k^2 + a^2 m_{eff}^2)u_k = 0, \quad (45)$$

where the effective mass squared is defined as

$$m_{eff}^2 = \frac{\dot{D}^2}{4D^2} - \frac{\ddot{D}}{2D} - \frac{3H\dot{D}}{2D}. \quad (46)$$

From the background solution (27), we can learn that in the far past  $\dot{\Pi} \sim \eta^{-1}$  while  $H \sim -\eta^{-2}$ . Using this asymptotic solution, we obtain the approximate forms for the parameters

$$\begin{aligned} D &\simeq \frac{2 + 4(F/2M)^{\frac{3}{2}}(M_p\eta)^{-2}}{[1 + 2(F/2M)^{\frac{3}{4}}(M_p\eta)^{-1}]^2}, \\ c_s^2 &\simeq \frac{1 + 2(F/2M)^{\frac{3}{4}}(M_p\eta)^{-1} - 2(F/2M)^{\frac{3}{2}}(M_p\eta)^{-2}}{3[1 + 2(F/2M)^{\frac{3}{2}}(M_p\eta)^{-2}]} \end{aligned} \quad (47)$$

In the far past when  $|\eta| \sim \sqrt{t_0 - t} \gg 1$ , and considering only the leading order terms, we obtain the explicit result

$$D \simeq 2, \quad c_s^2 \simeq \frac{1}{3}. \quad (48)$$

In this case, Eq. (45) describes a massless oscillator, and its solution is exactly

$$u_k(\eta) \simeq \frac{e^{ic_s k \eta}}{\sqrt{2c_s k}}. \quad (49)$$

Note that, during nearly all of the contracting phase our universe is radiation-dominated, with a background EoS of approximately  $1/3$  (recall Fig. 4). Therefore the semi-analytic result (47) is valid before the bouncing phase, and will not be much sensitive to the parameters  $F$  and  $M$ , since the terms containing these parameters are all negligible in the far past, as can be seen from Eq. (47). Thus, the perturbation variable will always oscillate since its effective mass squared vanishes. This behavior does not stop until the quintom scenario takes place and then the bouncing phase occurs. In this case, the cosmological perturbations seeded by the Galileon field in the primordial era are stable and form a blue spectrum as in the pure Galilean Genesis model. Obviously, this is incompatible with current observations.

In order to produce a scale-invariant primordial power spectrum in a Galileon bounce model, we are forced to consider the non-adiabatic modes of the cosmological perturbations. One possible mechanism, which was applied to a similar problem in Ref. [69], is to introduce a scalar field which plays the role of a curvaton in bouncing cosmologies. Provided that this curvaton field couples to the Galileon field in a suitable matter, we expect that its fluctuations will give rise to a scale-invariant power spectrum. We will discuss this issue in the next section. Before that, we turn our attention to tensor perturbations.

### B. Tensor perturbations

At leading order, the perturbed metric with respect to tensor perturbations is

$$ds^2 = -a^2(\eta)[d\eta^2 - (\delta_{ij} + \gamma_{ij})dx^i dx^j] \quad (50)$$

where  $\gamma_{ij}(x)$  is the tensor perturbation. Since in the Galileon model the higher derivative term of the Galileon field does not play a role in tensor perturbations [58], we can write down the following second order action for tensor perturbations

$$\mathcal{S}_2^T = \frac{1}{2} \int d\eta d^3x a^2 [\gamma_{ij}'^2 - (\partial_k \gamma_{ij})^2] . \quad (51)$$

By imposing the traceless and transverse conditions on  $\gamma_{ij}$ , its 9 components contain only two degrees of freedom. These may be expanded as usual [87]:

$$\gamma_{ij} = \sum_{\lambda=1}^2 \int \frac{d^3k}{(2\pi)^{3/2}} \gamma_{k,\lambda} e_{ij}^\lambda e^{ikx} . \quad (52)$$

After defining  $v_g^\lambda(k) \equiv a\gamma_{k,\lambda}$ , we obtain the equation of motion for the variable  $v_g^\lambda(k)$

$$v_g^\lambda(k)'' + (k^2 - \frac{a''}{a})v_g^\lambda(k) = 0 . \quad (53)$$

Since in the contracting phase the universe is radiation-dominated and thus  $a(\eta) \sim \eta$ , the last term in the above equation vanishes. This means, like the case of scalar cosmological perturbations, that there is only a massless oscillating solution:

$$v_g^\lambda(k) = \frac{e^{ik\eta}}{\sqrt{2k}} . \quad (54)$$

However, the tensor perturbation will always get frozen at a super-Hubble scale after the nonsingular bounce. In that case, from the above calculation, right after the expansion of the universe starts, the tensor perturbation will give rise to a blue-tilted spectrum:

$$\mathcal{P}^T(k) \equiv \frac{k^3}{2\pi^2} |v_g^\lambda(k)|^2 \sim k^2 , \quad (55)$$

with spectral index

$$n_T \equiv \frac{d \ln \mathcal{P}^T(k)}{d \ln k} = 2 . \quad (56)$$

Thus, for small  $k$  modes within observable scales, the amplitude of the tensor spectrum is severely suppressed. Correspondingly, the scalar-to-tensor ratio  $r \equiv \mathcal{P}^T/\mathcal{P}^S$  is expected to be extremely small, and so the tensor perturbations of Galileon bounce cosmologies will be very difficult to be detect in future CMB observations.

## V. THE BOUNCE CURVATON

A bounce curvaton scenario can be employed to generate a primordial power spectrum which is consistent with CMB observations [76]. It is known that entropy fluctuations are very important for the formation of the primordial power spectrum. In the context of inflationary cosmology, it has been realized that entropy fluctuations can lead to an additional source of curvature perturbations. This is now known as the curvaton mechanism. Entropy fluctuations also play an important role in the Ekpyrotic scenario [23, 24]. In that scenario, the primordial adiabatic fluctuations in the contracting phase acquire a deep blue spectrum, whereas the entropy modes induced by light scalar fields will be scale-invariant. Our model is similar to the case of Ekpyrotic scenario in that the adiabatic fluctuations seeded by the Galileon give rise to a blue spectrum. Thus, we expect that the bounce curvaton mechanism can contribute to the curvature perturbations so as to produce a scale-invariant power spectrum.

### A. The bounce curvaton scenario with kinetic coupling to the Galileon

We consider a massless curvaton field  $\sigma$  kinetically coupled to the background Galileon

$$\mathcal{S}_\sigma = \int d^4x \sqrt{-g} [-\Pi^q (\partial\sigma)^2] , \quad (57)$$

where  $q$  will be determined momentarily. A similar curvaton scenario was studied in Refs. [77–80]. A conformally-invariant coupling was considered and it was argued that a strong-coupling regime can be avoided [79, 80]. We would like to generalize this type of coupling term. We note that this coupling, like the coupling of the Galileon to gravity, never preserves the Galilean conformal symmetry<sup>4</sup>.

Working with conformal time, the equation of motion for a mode of the curvaton field with a fixed comoving wave number  $k$  takes the form

$$\sigma_k'' + (2\mathcal{H} + \frac{q\Pi'}{\Pi})\sigma_k' + k^2\sigma_k = 0 , \quad (58)$$

where  $\mathcal{H} = aH$  is the conformal Hubble parameter. From the analysis of background solution, we already know that the universe is initially radiation-dominated in the contracting phase, which implies that  $\Pi \sim a \sim \eta$ . Thus, we have  $\mathcal{H} \simeq 1/\eta$ , and so the equation of motion simplifies to

$$\sigma_k'' + \frac{2+q}{\eta}\sigma_k' + k^2\sigma_k = 0 . \quad (59)$$

The general solution to this equation can be expressed as a combination of two Bessel functions

$$\sigma_k = |\eta|^{-\frac{1+q}{2}} \left[ c_1 J_{-\frac{1+q}{2}}(k|\eta|) + c_2 J_{\frac{1+q}{2}}(k|\eta|) \right] . \quad (60)$$

Here  $c_1$  and  $c_2$  are two coefficients which may in principle depend on  $k$ . However, when we match the asymptotic behavior of the solution (60) in the limit of  $k|\eta| \gg 1$  with the sub-Hubble solution  $\sigma_k \rightarrow e^{ik\eta}/\sqrt{2k}$ , we find that both  $c_1$  and  $c_2$  are independent of the comoving wave number  $k$ . Consequently, the asymptotic behavior of (60) at super-Hubble scales is

$$\sigma_k \sim \tilde{c}_1 k^{-\frac{1+q}{2}} |\eta|^{-(1+q)} + \tilde{c}_2 k^{\frac{1+q}{2}} , \quad (61)$$

which consists of a constant mode and a varying (either decaying or growing) mode. Here  $\tilde{c}_i$  and  $c_i$  differ by an irrelevant factor.

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<sup>4</sup> The exponential coupling which preserves this symmetry yields a scale-invariant spectrum in the usual Galilean Genesis cosmology which is Minkowski in the infinite past, but not our cosmology, which begins in a radiation-dominated phase. This is because substituting our radiation-dominated contracting solution into the effective inverse metric  $\sqrt{g}g^{\mu\nu}e^{2\Pi}$  does not yield a fake de Sitter metric, however the  $\Pi^{-4}$  coupling to be described below does have this property.

### B. Scale-invariant power spectrum

We consider two cases:

i)  $q = 2$ : In this case, the super-Hubble solution for  $\sigma_k$  becomes

$$\sigma_k \sim \tilde{c}_1 k^{-\frac{3}{2}} |\eta|^{-3} + \tilde{c}_2 k^{\frac{3}{2}}. \quad (62)$$

Since in the radiation-dominated contracting phase  $|\eta|$  is a decreasing function with respect to cosmic time  $t$ , the first term of this solution is growing and thus will be dominant. Therefore the power-spectrum  $\mathcal{P}_\sigma$  will be

$$\begin{aligned} \mathcal{P}_\sigma &\equiv \frac{k^3}{2\pi^2} |\sigma|^2 \\ &\simeq \frac{\tilde{c}_1^2}{2\pi^2} |\eta|^{-6}, \end{aligned} \quad (63)$$

which is scale invariant and its amplitude is growing.

ii)  $q = -4$ : In this case, the super-Hubble solution of  $\sigma_k$  is

$$\sigma_k \sim \tilde{c}_1 k^{\frac{3}{2}} |\eta|^3 + \tilde{c}_2 k^{-\frac{3}{2}}. \quad (64)$$

For the same reason as above, here the first term is a decaying mode and thus the dominant mode is the last term. Thus, the power spectrum is

$$\mathcal{P}_\sigma \simeq \frac{\tilde{c}_2^2}{2\pi^2}, \quad (65)$$

which is also scale invariant and its amplitude is a constant.

In order to determine whether the curvaton mechanism is reliable, one has to estimate the backreaction of the curvaton field on background dynamics. From the action (57), one can easily find the effective energy density of the curvaton field  $\rho_\sigma = \Pi^q \dot{\sigma}^2$ . Solving the equation of motion (59) in the contracting phase, we obtain  $\sigma \sim (t_0 - t)^{-\frac{1+q}{2}}$ . Thus, the effective energy density of  $\sigma$  takes the form,

$$\rho_\sigma \sim (t_0 - t)^{-3-\frac{q}{2}}, \quad (66)$$

where we have applied the solution of  $\Pi$  in the radiation-dominated contracting phase. Recall that, since the universe shrinks in a radiation-dominated phase, the background energy density evolves as  $\rho \sim (t_0 - t)^{-2}$ . Therefore, a stable evolution for the curvaton field requires  $q \geq -2$ . As a consequence, we find the case of  $q = 2$  is a stable solution for the bounce curvaton scenario. However, in the case  $q = -4$ , although one obtains a scale-invariant spectrum of isocurvature perturbation, one needs to fine tune the initial condition of the curvaton field very carefully to prevent its backreaction from destroying the Galileon bounce.

Note that the fluctuations seeded by the curvaton field  $\sigma$  are mainly isocurvature modes in the contracting phase. However, they may easily be converted into curvature perturbations after the bounce. In this process, there are two important effects. First, for the adiabatic modes of cosmological perturbations in the contracting phase, the corresponding spectrum is blue tilted so that its amplitude is strongly suppressed at large length scales. Thus the main contribution to curvature perturbations at large length scales (corresponding to the observable scales) automatically come from the non-suppressed fluctuations of curvaton field. Second, after the bounce, it is expected that there exists a preheating phase during which the curvaton plays the role of an entropy field and reheats the universe [70] (see also [88] for an earlier study of the scenario of curvaton reheating). In this phase, the entropy fluctuations of the curvaton are transformed into curvature perturbations at all length scales.

## VI. CONCLUSIONS AND DISCUSSION

In this paper we have presented and studied bouncing solutions of the conformal Galileon model. The Galileon effective field description shows that the NEC can be violated safely without the usual ghost and gradient instabilities. Moreover, the terms in the Lagrangian leading to the violation of NEC are higher order operators, which are typically suppressed at low energy scales. Thus it is natural to consider NEC violation at high energy scales in a Galileon model, and if applied to cosmology, NEC violation usually occurs in the early universe. We study the conformal Galileon as a toy model. We find that in this model the universe can begin in a radiation-dominated contracting

phase. Along with the contraction, the EoS of the universe evolves from  $w = 1/3$  to a regime of Phantom phase  $w < -1$  and thus a quintom scenario is realized. After that, the universe passes through a nonsingular bouncing phase and finally approaches the moment of Galilean Genesis. We examined this solution both semi-analytically and numerically.

After having analyzed the background dynamics, we studied the perturbation theory of this model. It is well-known that, for a generic cosmological model described by a single k-essence field, the cosmological perturbations diverge in quintom scenarios [37]. In the past people were forced to consider models involving multiple degrees of freedom to circumvent this problem. However, in the Galileon model, although the number of degrees of freedom does not change, the stress energy tensor is in general that of an imperfect fluid in the sense that once one includes perturbations the viscosity is nonzero [89]. The novel behavior of this fluid allows it to avoid the usual divergence of perturbations when the EoS of the model crosses over the cosmological constant boundary. Our result confirms this conclusion and also shows that, for linear perturbations about Galileon bouncing cosmologies, there is no ghost instability and no superluminal propagation for linear perturbation modes. Therefore, in this background, the model is well-behaved and causal. However, our analysis shows that the adiabatic modes of cosmological perturbations yield a blue tilted spectrum, and so do tensor perturbations. In order to be consistent with the current cosmological observations, we include a bounce curvaton which couples to the Galileon field through its kinetic term. We obtained two such possible couplings which yield a scale-invariant power spectrum, with one possessing a growing spectrum in the contracting phase and the other a constant spectrum. The coupling to curvaton in both cases, however, breaks conformal symmetry explicitly. We examined the back reaction of curvaton on the background evolution, and find that in one case the bouncing solution is safe from the effect of curvaton, while in the other case fine-tuning might be needed.

At the end of the paper, we would like to comment on some issues. First, we ought to be aware of the final state of this model, i.e., the Galilean Genesis. A promising approach to avoiding strong coupling is to introduce a phase of preheating right before the moment of Galilean Genesis, as analyzed in Ref. [70]. Although the authors of [70] performed their analysis in the background of an emergent universe, their results can be roughly applied to bouncing cosmologies, providing an important supplement to the bounce preheating scenario [83]. Another interesting issue relates to the predictions of a Galileon bounce. From the study of linear perturbations, we predict that as in the original Galilean genesis proposal, primordial tensor perturbations will have a blue spectrum. This means that it will be difficult to observe the tensor spectrum in future CMB experiments. Obviously this is not a good news for this Galileon model of cosmology. However, we might have chance to provide experimental signatures for a Galileon bounce if we study nonlinear perturbations. Namely, it is known that, for a bouncing universe with an EoS of order unity in the contracting phase, the corresponding non-Gaussianities are quite sizable and might be sensitive to the forthcoming cosmological data [90]. Therefore, these non-Gaussianities in cosmological perturbations merit further study in a future project.

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*Note added*—After completing this manuscript we learned that a treatment of bouncing solutions in a more general set of theories has been done by Easson, Sawicki and Vikman, which will soon appear in Ref. [91].

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